

# Mark Scheme (Results)

## Summer 2017

Pearson Edexcel GCE In Mechanics M2 (6678/01)



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Summer 2017 Publications Code 6678\_01\_1706\_MS All the material in this publication is copyright © Pearson Education Ltd 2017 • All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

• Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

• Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

• There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

• All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

• Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

• When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

• Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

### **PEARSON EDEXCEL GCE MATHEMATICS**

#### **General Instructions for Marking**

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>`M' marks</u>

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

(i) should have the correct number of terms

(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

#### <u>`A' marks</u>

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

#### <u>'B' marks</u>

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

#### 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol  $\sqrt{}$  will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- **\*** The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

#### **General Principles for Mechanics Marking**

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
  - M(A) Taking moments about A.
  - N2L Newton's Second Law (Equation of Motion)
  - NEL Newton's Experimental Law (Newton's Law of Impact)
  - HL Hooke's Law
  - SHM Simple harmonic motion
  - PCLM Principle of conservation of linear momentum
  - RHS, LHS Right hand side, left hand side.

Q.	Scheme	Marks	Notes
1a	I = 0.5(2i+3j) - 0.5(4j)	M1	Impulse-momentum equation. Dimensionally correct. Condone subtraction in wrong order.
	$(=0.5(2\mathbf{i}-\mathbf{j}))$	A1	Correct unsimplified
	$\left(=0.5(2\mathbf{i}-\mathbf{j})\right)$ $\left \mathbf{I}\right  = \frac{1}{2}\sqrt{2^2 + 1^2}$	M1	Correct method for modulus. Follow their <b>I</b>
	$=\frac{1}{2}\sqrt{5}(=1.12)$ Ns	A1	1.1 or better (from correct solution only)
		(4)	
1b	$\tan^{-1}(\pm 2)$ or $\tan^{-1}(\pm \frac{1}{2})$ or $\tan \theta = \pm 2$ or $\tan \theta = \pm \frac{1}{2}$ or equivalent	M1	Correct method for a relevant angle. Follow their <b>I</b>
	Required angle = $117^{\circ}$ (116.6° or better)	A1	Accept 243° (2.03 rads)
		(2)	
1balt	4j 2i - j $\alpha$ 2i + 3j		
	$\cos \alpha = \frac{16 + 5 - 13}{2\sqrt{5}\sqrt{16}} = \frac{1}{\sqrt{5}}$	M1	
	Required angle = $117^{\circ}$ (116.6°)	A1	Accept 243°
		(2)	
		[6]	

Q.	Scheme	Marks	Notes
2a	Constant speed $\Rightarrow$ no acceleration. Driving force = 200+50+900g sin $\theta$ +150g sin $\theta$	M1	Equation of motion of the truck. All terms required & dimensionally correct. Condone sin/cos confusion and sign error(s)
	<b>0r</b> $D - T - 200 - 900g \sin \theta = 0$ <b>and</b> $T - 50 - 150g \sin \theta = 0$		
		A1	At most one error Allow for 2 separate equations including <i>T</i>
		A1	Correct unsimplified expression for the driving force (no <i>T</i> )
	$= 250 + 1050g \times \frac{1}{9} (= 1393.3333)$		$\left(\frac{4180}{3}\right)$
	$P = \left(250 + 1050g \times \frac{1}{9}\right) \times 15$	M1	Use of $P = Fv$ with their F Independent M1. Could appear in first equation as $F = \frac{P}{v}$ .
	= 20900  W(20.9  kW)	A1	Accept 21000 W, 21kW. Maximum 3 s.f.
		(5)	
2b	$\left(their 1393\frac{1}{3}\right) - 200 - 900g \times \frac{1}{9} = 900a$	M1	Equation of motion for the truck at instant after the towbar breaks. All terms required & dimensionally correct. Allow for an equation to find acceleration down the slope
		A1ft	Correct for their driving force $\left(1393\frac{1}{3}\right)$ .
	$a = 0.237 \mathrm{m \ s^{-2}}$	A1	Accept 0.24, not $\frac{32}{135}$ must be +ve
		(3)	
2c	$\frac{1}{2} \times 150 \times 15^2 = 50d + 150g\sin\theta d$	M1	Must be using work-energy (for trailer only) All terms required & dimensionally correct. Condone sin/cos confusion and sign error(s)
		A1	Unsimplified equation with at most one error
	$\left(16875 = 50d + \frac{150}{9}gd\right)$	A1	Correct unsimplified equation for <i>d</i>
	$d = 79 \mathrm{m}$ (79.1)	A1	Maximum 3 s.f.
		(4)	
		[12]	
		1	

Q.	Scheme						Marks	Notes
		FGD E	2 of CFG	CBAG	L			
3a	Mass ratio	4	4	4	12		B1	Mass ratios
Ja	C of M from EF	а	$\frac{4}{3}a$	3a	d		B1	Distances from <i>EF</i> or an alternative vertical axis
	$12d = 4a + 4 \times \frac{4}{3}a + 4 \times 3a$						M1	Moments about <i>EF</i> or equivalent Need all terms and dimensionally correct
							A1	Correct unsimplified equation
	120	d = 16a	$\times \frac{4}{3}, d =$	$\frac{16}{9}a$			A1	Sufficient working to justify <b>*given</b> answer*
							(5)	
3a alt	Splitting the rectangle into a pair of trapeziums gives mass ratios 1: 1: 2					ms	B1	
	Distance of c of m of ABCF is $\frac{8}{9}a$ from AF and $\frac{22}{9}a$ from EF						B1	
	$2a - a \times \frac{8}{9} + a \times \frac{22}{9} \left( = \frac{32}{9}a \right) = 2d$						M1A1	
	$d = \frac{16}{9}a$						A1	
							(5)	
	Square -sc					,		
		L sq	S sq	-tri +tr	i I			
3a alt	mass	4	1	$\frac{1}{2}$ $\frac{1}{2}$	3	3	B1	
	From EF	2a	3a	$\frac{2a}{3}$ $\frac{4a}{3}$	- 0	l	B1	
	$4 \times 2a - 3a$	$a - \frac{1}{2} \times \frac{2}{3}$	$\frac{2a}{3} + \frac{1}{2} \times \frac{4}{2}$	$\frac{4a}{3} \left( = \frac{16a}{3} \right)$	=3d		M1A1	
	$d = \frac{16}{9}a$				!		A1	
							(5)	

Q.	Scheme	Marks	Notes
3b	$F = \frac{\frac{16}{9}a}{G} = \frac{16}{G}$		
	Symmetry $\Rightarrow$ c of m $\frac{16}{9}a$ from C	B1	For vertical distance – allow for $\frac{20}{9}a$ or equivalent
	$\tan^{-1}\frac{1}{8}$ ( $\tan^{-1}8$ )	M1	Correct trig to find relevant angle (using $\frac{2}{9}a$ horizontally and their vertical $\neq 2a$ )
	7.125°	A1	(7.1°,82.9°,0.124rads,1.45rads)
	$\theta = 37.9^{\circ}$	A1 (4)	38° or better (37.874°, 0.66 rads)
3b alt	$F$ $C$ $C$ $\theta$ $y$ $y$ $c$ $X$ $G$		Using cosine rule With $x = \frac{16}{9}\sqrt{2}a$ , $c = \frac{14}{9}a$ $y = \sqrt{65} \times \frac{2a}{9}$
	Symmetry $\Rightarrow$ c of m $\frac{2}{9}a$ from FG	B1	
	$\cos\theta = \frac{x^2 + y^2 - c^2}{2xy} = \frac{9}{\sqrt{130}} = 0.789$	M1A1	
3balt	$\theta = 37.9^{\circ}$	A1 (4)	
	Symmetry $\Rightarrow$ c of m $\frac{2}{9}a$ from FG	B1	i.e. on bisector of angle G
	$\tan \theta = \frac{MX}{CX} = \frac{\sqrt{2}a - \frac{2\sqrt{2}}{9}a}{\sqrt{2}a} = \frac{7}{9}$	M1A1	Using Isosceles triangles
	$\theta = 37.9^{\circ}$	A1 (4)	
		[9]	

Q.	Schomo	Marks	Notos
	Scheme		Notes
<b>4</b> a	$v = 0 \Longrightarrow 3t^2 - 16t + 21 = 0$	M1	Set $v = 0$ and attempt to solve
	$v = 0 \Longrightarrow 3t^{2} - 16t + 21 = 0$ ((3t-7)(t-3)=0) $t_{1} = \frac{7}{3}, t_{2} = 3$	A1	
		(2)	
<b>4</b> b	$a = \frac{\mathrm{d}}{\mathrm{d}t} \left( 3t^2 - 16t + 21 \right)$	M1	Differentiate v to obtain a
	= 6t - 16	A1	
	= 6t - 16 $t = t_1,  a = 6 \times \frac{7}{3} - 16 = -2 \text{ (m s}^{-2})$ Magnitude 2 (m s <sup>-2</sup> )	A1	No errors seen. Must be positive - the Q asks for magnitude.
		(3)	
	$\left( \left( \begin{array}{c} 2 \\ - \end{array} \right) \right)$		
<b>4</b> c	$s = \int \left(3t^2 - 16t + 21\right) \mathrm{d}t$	M1	Integrate <i>v</i> to find <i>s</i>
	$=t^{3}-8t^{2}+21t(+C)$	A1	
	$\pm \left( \left( 3^3 - 8 \times 9 + 21 \times 3 \right) - \left( \left( \frac{7}{3} \right)^3 - 8 \times \frac{49}{9} + 21 \times \frac{7}{3} \right) \right)$	M1	Correct use of their limits
	$s = 0.148 \text{ (m)} \left(\frac{4}{27}\right)$	A1	Final answer must be positive. 0.15 or better
		(4)	
<b>4d</b>	Return to $O \Longrightarrow s = 0 = t(t^2 - 8t + 21)$	B1	seen or implied
	Discriminant of quadratic = $64 - 4 \times 21(=-20) < 0$	M1	Or equivalent. *given answer so must show some evidence of method*
	No real roots $\Rightarrow$ does not return to $O$	A1	Sufficient correct working to justify <b>*given</b> answer*
		(3)	
4dalt	Travels away until $t_1 = \frac{7}{3}$ , turns back at $t_2 = 3$ then turns away again	M1	Complete story
	$s_3 = 18$	B1	Seen or implied
	Complete argument	Al	· · · · ·
		(3)	
4dalt	Distance time graph	B1	
	Locate min turning point	M1	
	Complete argument	A1	
		(3)	
		[12]	

Q.	Scheme	Marks	Notes
5a	$A \xrightarrow{1 \text{ m } C} \beta \xrightarrow{\beta} 5g \xrightarrow{4 \text{ m } \alpha} B \xrightarrow{\frac{2}{3}R}$		
	$F = \frac{2}{3}R$ seen or implied	B1	Use of $F = \mu R$ . Could be on diagram. Allow in (b) if not seen before
	$M(C): 5g \times 3\cos\alpha + F \times 7\sin\alpha = 7\cos\alpha \times R$	M1	Moments about <i>C</i> or alternative complete method to find equation in <i>F</i> and <i>R</i> or <i>R</i> only. Dimensionally correct and all terms needed. Condone sin/cos confusion and sign error(s).
		A1	At most one error
		A1	Correct unsimplified equation
	$15g\cos\alpha = R\left(7\cos\alpha - \frac{14}{3}\sin\alpha\right)$		
	$15g \times \frac{4}{5} = R\left(7 \times \frac{4}{5} - \frac{14}{3} \times \frac{3}{5}\right) = \frac{14}{5}R$	dM1	Substitute for <i>F</i> and trig and solve for <i>R</i> Dependent on previous M1
	$R = \frac{30}{7}g = 42$ (N)	A1	
		(6)	
	e.g. of alternative for M1A1A1:		
	M(A): $T \sin \beta + 8R \cos \alpha = 8F \sin \alpha + 20g \cos \alpha$ and M(B): $7T \sin \beta = 20g \cos \alpha$	(M1)	
		(A1)	At most 1 error
	$\frac{20g}{7}\cos\alpha + 8R\cos\alpha = 8F\sin\alpha + 20g\cos\alpha$	(A1)	Correct unsimplified equation in <i>F</i> and <i>R</i> or <i>R</i> only
		1	

Q.	Scheme	Marks	Notes
5b	Resolve $\updownarrow$ : $T \cos \theta + R = 5g$ $R + T \sin(\beta - \alpha) = 5g$	M1	Need all terms. Condone sin/cos confusion and sign error(s).
		A1	Correct in <i>R</i> or <i>their R</i>
	Resolve $\leftrightarrow$ : $T\sin\theta = F(=28)$ $F\left(=\frac{2}{3}R\right) = T\cos(\beta - \alpha)$	M1	Need both terms. Condone sin/cos confusion
		A1	Correct in <i>R</i> or <i>their R</i>
	Solve simultaneous equations for $\beta - \alpha$		
	$\tan(\beta - \alpha) = 4, \ \beta = 50.9^{\circ}  (51^{\circ})$	A1	cso. Max 3 s.f.
		(5)	
Alt 5b	$M(B): 7 \times T \sin \beta = 5g \cos \alpha \times 4$	M1	Moments equation. Dimensionally correct. Condone sin/cos confusion and sign error(s).
	$\left(T\sin\beta = \frac{16}{7}g\right)$	A1	
	OR: resolve perpendicular to the rod: $T \sin \beta + R \cos \alpha = 5g \cos \alpha + \frac{2}{3}R \sin \alpha$	(M1) (A1)	
	Resolve parallel to rod: $T \cos \beta + 5g \sin \alpha = F \cos \alpha + R \sin \alpha$ $\left( = \frac{2}{3}R \cos \alpha + R \sin \alpha \right)$	M1	All terms needed. Condone sin/cos confusion and sign error(s).
	$\left(T\cos\beta = \frac{13}{7}g\right)$	A1	
	Solve simultaneous equations for $\beta$		
	$\tan \beta = \frac{16}{13}$ , $\beta = 50.9^{\circ}$ (51°)	A1	cso. Max 3 s.f.
		(5) [11]	

Q.	Scheme	Marks	Notes
6a	$\begin{array}{c} 30 \text{ ms}^{-1} & P \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & & \\ \hline & & & &$		
	$30\cos 60 \times 2 + q\cos \theta \times 2 = 40$	M1	Equation for horizontal distance Need to be using the 40 m
		A1	Correct unsimplified
	$30\sin 60 \times 2 - 4.9 \times 4 = q\sin\theta \times 2 - 4.9 \times 4$ $30\sin 60 = q\sin\theta$	M1	Equal vertical distance or initial vertical components of velocity
		A1	Correct unsimplified (no error seen)
	$q\cos\theta = \pm 5$		
	$q\sin\theta = 15\sqrt{3}$		
	$\tan\theta = 3\sqrt{3}$		Solve for $q$ or $\theta$
	$(\tan\theta = 6\sin 60)$	DM1	Dependent on both preceding M marks
	$\theta = 79.1$ (79)		(1.38 radians) or better
	q = 26.45 = 26.5	A1	(26 or better) $(10\sqrt{7})$ Both correct and no error seen
		(6)	
		(0)	
6b	Vertical component of speed =	M1	Must be working towards speed of $P$ (or $v^2$ ) (condone if working on $Q$ - they equal vertical components of velocity)
	$30\sin 60 - 2g(=6.38)$	A1	Correct unsimplified. Accept ±
	speed = $\sqrt{(30\cos 60)^2 + 6.38^2}$	DM1	Use Pythagoras. Dependent on previous M Follow their vertical component.
		Alft	Correct unsimplified equation in $v$ or $v^2$ .
	$=\sqrt{15^2+6.38^2}=16.3 ({\rm m \ s^{-1}})$	A1	or 16 2 or 3 sf only
		(5)	
6b alt	Vertical distance =	M1	Must be working towards speed of P
	$30\sin 60 \times 2 - 4.9 \times 4 = 32.36$	A1	Correct unsimplified
	Conservation of energy:	DM1	Dependent on previous M. Follow their vertical distance.
	$\frac{1}{2}mv^2 + mg \times 32.36 = \frac{1}{2}m \times 900$	A1ft	Correct unsimplified equation in $v$ or $v^2$ .
	$v = 16.3 \text{ (m s}^{-1}) (16)$	A1	
		(5)	
		[11]	

Q.	Scheme	Marks	Notes
	Contente	Marks	10103
7a	$ \begin{array}{c} A \\ 3m \\ \leftarrow x \end{array} \qquad \begin{array}{c} & & & \\ B \\ 4m \\ & & \\ &$		
	Impact with wall: $v = \frac{3}{5} \times 5u = 3u$	B1	or $-3u$
	Impulse $\pm 4m(3u-(-5u))$	M1	M0 if clearly using $mv + mu$ , otherwise bod
	Magnitude 32mu (Ns)	A1	
		(3)	
7b	CLM: $3mx + 4mw = 4m \times 3u$	M1	Need all 4 terms. Condone sign errors. Use of $5u$ is M0
		A1ft	follow their 3 <i>u</i>
	Impact: $x - w = e \times 3u$	M1	Used the right way round. Use of $5u$ is M0
		A1ft	follow their 3 <i>u</i>
			signs consistent with CLM equation
	3m(w+3eu)+4mw=7mw+9emu=12mu		
		DM1	Solve for <i>w</i> or <i>kw</i> .
	7w = u(12 - 9e)		Dependent on two preceding M marks
	Use of $e \le 1$ in their w: $7w \ge 3u$	M1	Condone use of <
	Hence $w > 0$ and A and B are moving in the same direction	A1 (7)	Complete argument leading to <b>*given</b> answer*
	KE of <i>B</i> before collision	, í	
7c	$=\frac{1}{2}\times 4m\times (3u)^2 (=18mu^2)$	B1	follow their 3 <i>u</i> . seen or implied
	$\Rightarrow \frac{1}{2} \times 4m \left( \frac{u}{7} (12 - 9e) \right)^2 = \frac{1}{4} \left( \frac{1}{2} \times 4m \times 9u^2 \right)$	M1	Follow their w. $\frac{1}{4}$ on the right side.
	$4(12-9e)^2 = 49 \times 9, \ (4-3e)^2 = \frac{49}{4}$	A1	Correct equation in <i>m</i> , <i>u</i> and <i>e</i>
	$e = \frac{1}{6}$	A1 (4)	
	KE of <i>B</i> before collision		follow their 3 <i>u</i>
7c alt	$=\frac{1}{2}\times 4m\times (3u)^2 (=18mu^2)$	B1	
	$\Rightarrow \frac{1}{2} 4mw^2 = \frac{1}{4} \times \frac{1}{2} \times 4m(3u)^2 \qquad \left(w = \frac{1}{2} \times 3u\right)$	M1	$\frac{1}{4}$ on the right side.
	$\frac{3}{7}(4-3e) = \frac{1}{2} \times 3$	A1	Correct equation in <i>m</i> , <i>u</i> and <i>e</i> from correct work only
	$e = \frac{1}{6}$	A1 (4)	0.17 or better from correct work only
	~~	[14]	
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